CDI-II - Prática F.3 17/3/21

$$\frac{d}{dx} f(u(x))\Big|_{x=a} = f'(u(a))\, u'(a)$$

$$\begin{array}{ccccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R} & \longrightarrow & \mathbb{R} \\ a & \longmapsto & u(a) & \longmapsto & f(u(a)) \end{array}$$

1-a) $\frac{\partial f}{\partial x}(x,y) = \frac{2x}{x^2+y^2}$

etc.

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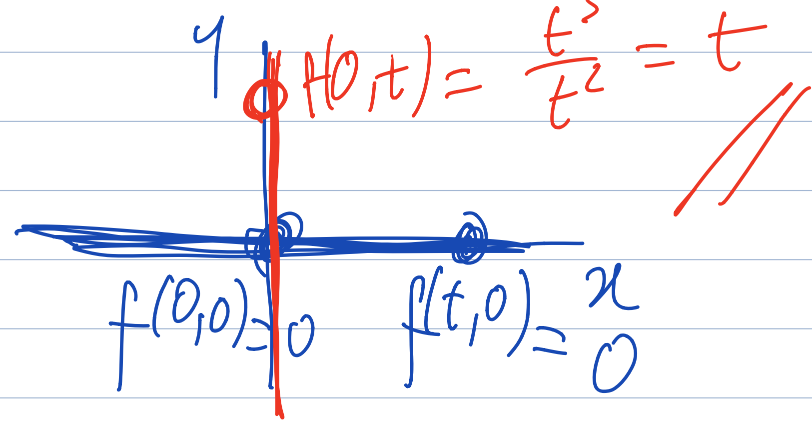
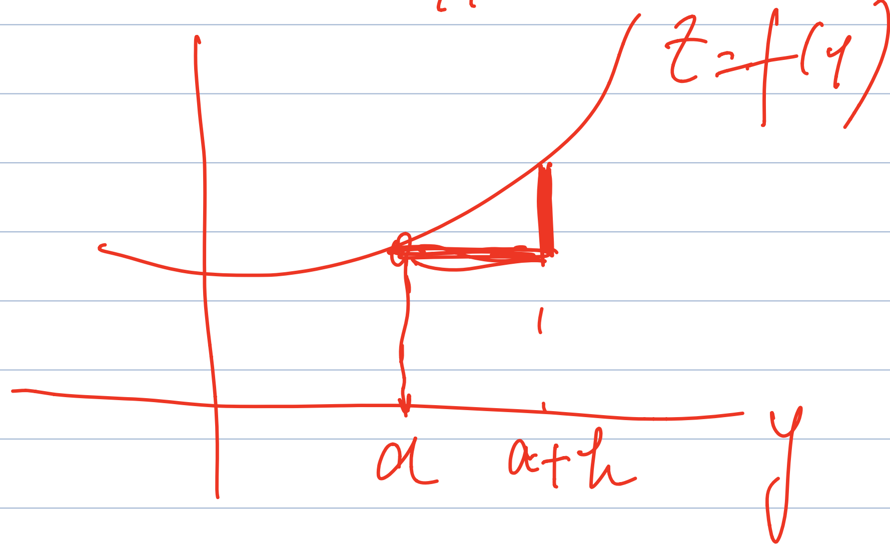
2 -



$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0} \frac{f(t,0)-f(0,0)}{t} = 0$$

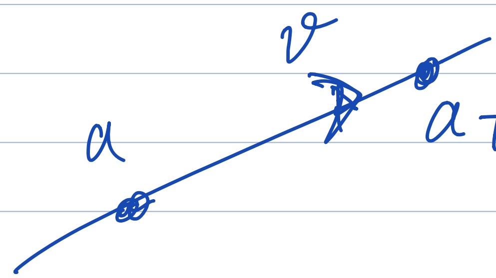
$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0} \frac{f(0,t)-f(0,0)}{t} = \lim_{t\to 0} \frac{t}{t} = 1$$



3-e) $D\gamma(t) = \gamma'(t) = \begin{bmatrix} 3t^2 \\ -e^{-t} \\ -\frac{1}{t^2} \end{bmatrix}$

etc.

4 - $\frac{\partial f}{\partial v}(a) = \lim_{t \to 0} \frac{f(a+tv)-f(a)}{t}$



$a+tv$, $t \in \mathbb{R}$

Se $\underline{f \text{ for diferenciável}}$ em a então

$$\frac{\partial f}{\partial v}(a) = \underbrace{Df(a)\,v}$$

4-a) y^x, $y>0$

$$y^x = e^{x \ln y}$$

$$f(x,y) = e^{x \ln y}$$

etc.

5- $0 = \dfrac{\partial f}{\partial v}(a) = Df(a)\,v$

?

$a = (1,2)$

$f(x,y) = xy^2 + x^2y \qquad v = (v_1, v_2)\,?$

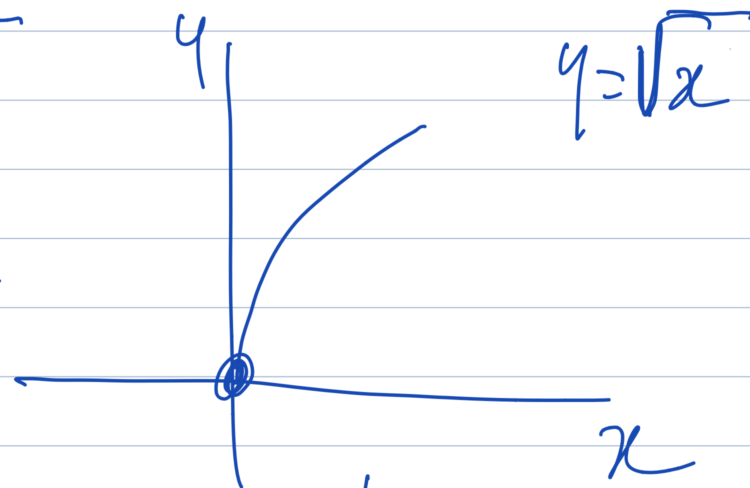
$Df(x,y) = \begin{bmatrix} y^2+2xy & 2xy+x^2 \end{bmatrix}$

$Df(1,2) = \begin{bmatrix} 8 & 5 \end{bmatrix}$

$0 = 8v_1 + 5v_2$

$(v_1, v_2) = (5, -8)$

6. $\sqrt{\cdot}$



não é dif.
na origem.

$\Downarrow$

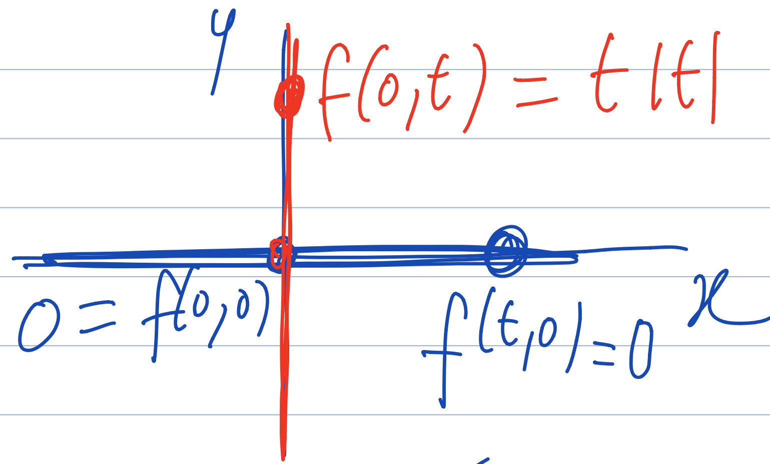
usar a definição!

$$f(a+h) - f(a) - \underbrace{Df(a)h}_{\text{derivadas parciais}} = o(h)$$

$a = (0,0)$

$$f(x,y) = y\sqrt{x^2+y^2} = y\,\|(x,y)\|$$

1- Calcular derivadas parciais em $(0,0)$.



$$\frac{\partial f}{\partial x}(0,0) = 0 \,. \quad ✓$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t \to 0} \frac{f(0,t) - f(0,0)}{t}$$

$$= \lim_{t \to 0} \frac{\cancel{t}\,|t|}{\cancel{t}} = 0 \quad ✓$$

2 - $a = (0,0)$, $h = (x,y)$

$a + h = (x,y)$

$$f(x,y) - \underbrace{f(0,0)}_{=0} - \underbrace{Df(0,0)}_{[0\ 0]\begin{bmatrix} x \\ y \end{bmatrix} = 0}(x,y) \overset{?}{=} o(x,y)$$

$$f(x,y) \cancel{\overset{?}{=} o(x,y)}$$

a verificar!

$$\lim_{(x,y)\to(0,0)} \frac{|f(x,y)|}{\|(x,y)\|} = 0 \;?$$

$$\frac{|f(x,y)|}{\|(x,y)\|} = \frac{|y|\,\cancel{\|(x,y)\|}}{\cancel{\|(x,y)\|}}$$

$$= |y| \longrightarrow 0$$

f é dif. em $(0,0)$ e

$$Df(0,0) = \begin{bmatrix} 0 & 0 \end{bmatrix}.\;!$$

7 - 1) Se f for dif. em a então f é contínua em a.

2) Se f for dif. em a, então

$$\frac{\partial f}{\partial v}(a) = Df(a)\,v$$

3) f é dif. se

$$f(a+h) - f(a) - \underbrace{Df(a)}_{\text{matriz das derivadas parciais}} h = o(h)$$

7-i) f não é contínua em $(0,0)$:

$$f(x,0) = 1 \qquad f(0,0)=0$$

f não é diferenciável em $(0,0)$.

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7(ii) $$0 \le \left|\frac{xy}{\|(x,y)\|}\right| = \frac{|x||y|}{\|(x,y)\|} \le |y| \to 0$$

$$\le 1$$

g é contínua em $(0,0)$

$$\frac{\partial g}{\partial x}(0,0) = 0 \;, \quad \frac{\partial g}{\partial y}(0,0) = 0$$

$$g(0,0) = 0$$

$$g(x,y) - \underbrace{g(0,0)}_{=0} - \underbrace{Dg(0,0)\binom{x}{y}}_{=0} \overset{?}{=} o(x,y)$$

$$\left| \frac{g(x,y)}{\|(x,y)\|} \right| \xrightarrow[(x,y)\to(0,0)]{?} 0$$

$$\frac{\dfrac{|x|\,|y|}{\|(x,y)\|}}{\|(x,y)\|} = \frac{|x|\,|y|}{\|(x,y)\|^2} \quad ②$$

② Justificar: o limite não é zero

$y = x \qquad \dfrac{x^2}{2x^2} = \dfrac{1}{2} \neq 0$

$\|(x,x)\|^2 = x^2 + x^2 = 2x^2$

$\therefore$ g não é diferenciável em $(0,0)$.

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f iii) $\qquad \dfrac{x^2 y^2}{x^4 + y^2} \longrightarrow 4$

$\qquad\qquad\qquad\qquad\quad \longrightarrow 2$

x^{1000} $\qquad \leq 1$

suspeitar que é dif. em $(0,0)$. $\qquad y^2 \leq y^2 + x^4$

$$\frac{\partial h}{\partial x}(0,0) = 0 \quad , \quad \frac{\partial h}{\partial y}(0,0) = 0$$

$$h(0,0) = 0$$

$$h(x,y) - \cancel{h(0,0)} - \cancel{Dh(0,0)(x,y)} = o(x,y)$$

?

$$\left| \frac{h(x,y)}{\|(x,y)\|} \right| = \frac{\frac{x^2 y^2}{x^4+y^2}}{\|(x,y)\|} = \frac{\overset{|x||x|}{x^2}\, y^2}{\|(x,y)\|(x^4+y^2)}$$

≤ 1 ≤ 1

$$\leq |x| \longrightarrow 0$$

8 — a) $(0,1) \neq (0,0)$

$\Rightarrow$ usar as regras de derivação.

b) $Df(0,1)\begin{bmatrix}2\\1\end{bmatrix} = \ldots$ etc

$$\begin{bmatrix}\frac{\partial f}{\partial x}(0,1) & \frac{\partial f}{\partial y}(0,1)\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \ldots$$

c) $$\frac{\partial f}{\partial v}(a) = \lim_{t \to 0} \frac{f(a+tv)-f(a)}{t}$$

Em $(0,0)$ temos de usar a definição de $\frac{\partial f}{\partial v}(0,0)$.

$v = (2,3)$ $\quad a = (0,0)$

$a + tv = (2t, 3t)$

$$\frac{\partial f}{\partial v}(0,0) = \lim_{t \to 0} \frac{f(2t,3t) - f(0,0)}{t}$$

$$= \lim_{t \to 0} \frac{\frac{2t\,(3t)^2}{(2t)^2 + (3t)^2}}{t}$$

$$= \lim_{t \to 0} \frac{18\cancel{t^3}}{13\cancel{t^3}} = \frac{18}{13}$$

Note: $\frac{\partial f}{\partial x}(0,0) = 0$; $\frac{\partial f}{\partial y}(0,0) = 0$

$$Df(0,0)(2,3) = 0 \, !$$

$$\neq$$

$$\frac{\partial f}{\partial v}(0,0) = \frac{18}{13}$$

$v = (2,3)$.

$\Rightarrow$ f não é dif. em $(0,0)$.

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Exercício → em 7ii)
fazer o mesmo que em
8-c).

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